# DOES ALLAN VARIANCE DETERMINE THE SPECTRUM?

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The spectral density of phase or frequency is known to determine Allan variance, Avar($\tau$), by a straightforward frequency-domain integral, which maps a power-law spectrum (within a certain range of exponents) to a power-law Avar. Recognizing power-law Avar components in the log-log $\sigma$-$\tau$ plot, one infers the presence of the corresponding spectral components, thus performing a *parametric* inversion of the spectrum-to-Avar mapping. In the literature one sometimes sees claims of *general* inversion formulas that go direct] y from Avar to spectrum, or claims that Avar determines the spectrum *uniquely*. It is the latter claims that I show to be false. It follows that any general inversion formula, if it works at all, has to be one-sided, unless the class of spectra is restricted (see below).

The ambiguity of inversion is centered at flicker FM, whose Avar is constant. ('l'he case of white PM vs. flicker PM is not real l y an example.) 'l'here is an infinite set of spectra that also have constant Avar; I shall show how to construct a general example, and prove directly that a certain bright-line spectrum has constant Avar. A corol lary of the theorem about spectra with constant Avar is a characterization of the spectral ambiguity of Avar: *two FM spectra have the same A var if and only if they have the same power in every octave*. One can prove that an ambiguity occurs only for nonstationary FM noises; Avar does determine the spectrum uniquely over the class of stationary FM noises. Here, though, the spectrum is determined by a simpler time-domain function, the variance of the $\tau$-average frequency.

1 argue that these results deserve to be known by our community, even though the except ional spectra that constitute the ambiguity appear to be physical l y irrelevant. The results do imply that Avar should not be regarded as a "structure function", because it does not fully determine the covariance structure of the second differences of phase. Although Avar processing remains a convenient way to detect broad spectral trends, one cannot expect to extract spectral details by this technique.

This work was performed at the Jet Propulsion Laboratory, California Institute of '1'ethnology, under a contract with the National Aeronautics and Space Administration.

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The ambiguity of inversion is centered at flicker FM, whose Avar is constant. (The case of white PM vs. flicker PM is not really an example.) There is an infinite set of spectra that also have constant Avar; I shall show how to construct a general example, and prove directly that a certain bright-line spectrum has constant Avar. A corollary of the theorem about spectra with constant Avar is a characterization of the spectral ambiguity of Avar: *two FM spectra have the same Avar if and only if they have the same power in every octave.* One can prove that an ambiguity occurs only for nonstationary FM noises; Avar does determine the spectrum uniquely over the class of stationary FM noises. Here, though, the spectrum is determined by a simpler time-domain function, the variance of the z-average frequency.

I argue that these results deserve to be known by our community, even though the exceptional spectra that constitute the ambiguity appear to be physically irrelevant. The results do imply that Avar should not be regarded as a "structure function", because it does not fully determine the covariance structure of the second differences of phase. Although Avar processing remains a convenient way to detect broad spectral trends, one cannot expect to extract spectral details by this technique.

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